



# **MATERIAL BALANCE**

English – MSc in Petroleum Engineering MFKOT730026

## Course Description

Olaj- és gázmérnöki mesterszak  
MSc Angol Olaj

Nappali munkarend  
Daily work schedule

TANTÁRGYI KOMMUNIKÁCIÓS DOSSZIÉ  
CURRICULUM COMMUNICATION FILE

Miskolci Egyetem  
Műszaki Földtudományi Kar  
Kőolaj és Földgáz Intézet

Miskolc University  
Faculty of Earth Science and Engineering  
Institute of Petroleum and Natural Gas

Miskolc, 2022/2023 I. félév / Semester

*A tantárgy adatlapja / Course data sheet*

<b>Course Title:</b> Material Balance	<b>Code:</b> MFKOT730026
<b>Instructor:</b> Dr. Dmour Hazim Nayel AB., Associates Professor	<b>Responsible department/institute:</b> DPE/IPNG(OMTSZ/KFGI)
	<b>Course element:</b> Compulsory
<b>Position in curriculum*</b> <b>(Which semester):</b> 3 (2)	<b>Pre-requisites (if any):</b> Reservoir Engineering Fundamentals (MFKOT720024)
<b>No. of contact hours per week (lecture + seminar):</b> 2+1	<b>Type of Assessment (examination / practical mark / other):</b> examination
<b>Credits:</b> 3	<b>Course:</b> full time

**Task and purpose of the subject:**

To enable the students to understand how to maximize recovery by applying external energy sources.

**Competencies to evolve:****Knowledge:**

T1 Knows the economic processes related to the hydrocarbon industry.

T6 Familiar with the characteristics of the fluids found in the petroleum, natural gas, geothermal reservoirs and the properties of the rocks and the characteristics of the flow in porous mediums.

T7 Familiar with the production mechanisms of underground reservoirs and the primary or enhanced recovery methods for optimum output.

T8 Knows the basics of numerical simulation of underground reservoirs.

T11 Familiar with the methods and software of computer design and analysis in the hydrocarbon industry.

**Ability:**

K1 Able to interpret the economic processes related to the hydrocarbon industry and give adequate answers to them.

K6 Capable of forecasting the behavior of the fluids found in the petroleum, natural gas and geothermal reservoirs, the properties of the reservoir rocks and the characteristics of the seepage in the reservoirs.

K7 Able to recognize the production mechanisms of underground reservoirs and to select the optimum of primary or enhanced recovery mechanisms.

K8 Able to perform a numerical simulation of underground reservoirs.

K11 Capable to perform computer design and evaluations for hydrocarbon industry.

**Attitude:**

A1 Enforce sustainability and energy efficiency requirements.

A2 Strive professionally at a high level, independently or in a workgroup to plan and carry out tasks.

A3 Strives to carry out work using a complex approach based on a systematic and process-oriented mindset.

A4 Seeks to achieve research, development and innovation goals during work.

**Autonomy and responsibility:** F1, F4, F6, F7

F1 Independently capable of manage a hydrocarbon industrial complex design work and the task of performing and participating in Project manager tasks.

F4 Autonomously capable to choose the recovery mechanism of underground reservoirs; to select the most favorable reservoir management.

F6 Has an autonomous capacity to plan the use of renewable natural resources and from residues into the energy supply system, to operate the established system

F7 Takes responsibility for professional decisions, for carrying out workflows or managing them.

**Assessment and grading:**

Students will be assessed with using the following elements.

Attendance: 5 %, Homework 10%, Midterm exam 40 %, Final exam 45 %, Total 100%

It is possible to make up for an unsuccessful or unwritten midterm exam in the last class of the semester.

**Grading scale:**

90-100%: (5) excellent, 80-89%: (4) good, 70-79%: (3) satisfactory, 60-69%: (2) pass, 0-59%: (1) failed

**Compulsory or recommended literature resources:**

J. Pápay: Development of Petroleum Reservoirs, Akadémiai Kiadó, Budapest 2003. ISBN 963 05 7927 8.

Craft and Hawkins: Applied Petroleum Reservoir Engineering, Prentice Hall, 1991, ISBN 0-13-039884-5.

Towler: Fundamental Principles of Reservoir Engineering, SPE Textbook Series, Vol.8., 2002, ISBN 1-55563-092-8

T. Ahmed: Advanced Reservoir Engineering, Gulf Publishing Co. 2005, ISBN-13: 978-0-7506-7733-2.

T. Ahmed: Reservoir Engineering Handbook, Gulf Publishing Co., 2001, ISBN 0-88415-770-9.

L.P.Dake: Fundamentals of Reservoir Engineering, Elsevier, 1978, ISBN 0-444-41830-X

*Féléves ütemterv/ Course schedule*

<b>Dátum/ Date</b>	<b>Hét/ week</b>	<b>Téma/ Topic</b>
2020.09.09.	1.	Basic terms, conditions. Different forms of the material balance equation.
2020.09.16.	2.	Material Balance of the saturated oil reservoir and under saturated oil reservoir.
2020.09.23.	3.	Average pressure. Drive mechanics, drive indices
2020.09.30.	4.	Material balance equation of a gas reservoir.
2020.10.07	5.	Water influx.
2020.10.14	6.	Exam no.1
2020.10.21.	7.	Volumetric and open reservoirs
2020.10.28.	8.	Educational break
2020.11.04.	9.	Van Everdingen and Hurst method for water influx determination. Fetkovich method for water influx determination.
2020.11.11.	10.	Plots used to determine OOIP and OGIP for gas and oil reservoirs.
2020.11.18.	11.	Hydrocarbon in Place Estimation with material balance
2020.11.25.	12.	Exam no.2
2020.12.02.	13.	Havlena-Odeh, Schilthuis, Tarner, Tehrani, Sills methods.
2020.12.09.	14.	Prediction with material balance End of semester, writing supplementary tests

## Sample of Midterm Exam no.1:

### Question 1

For a given data:

Data

Time, days	Pe psi		
0	2740	Pi =	2740 psi
365	2500	Ar =	40363 acres
730	2290	Aa =	1000000 acres
1095	2109	h =	100 ft
1460	1949	k =	200 md
		ct =	7.00E-06 psi <sup>-1</sup>
		μw =	0.55 md
		θ =	140°

Step 1. Calculate the reservoir radius  $r_e$

Step 2. Calculate the equivalent aquifer radius  $r_a$

Step 3. Calculate the dimensionless radius  $r_D$ .

$$r_D = r_a / r_e$$

Step 4. Calculate initial water in place  $W_i$ .

$$W_i = \pi (r_a^2 - r_e^2) h \phi / 5.615$$

Step 5. Calculate  $(W_e)_{\max}$

$$(W_e)_{\max} = c_t W_i p_i f \quad \boxed{\phantom{0000}}$$

Step 6. Calculate the productivity index  $J$  of the radial aquifer

$$J = \frac{0.00708 kh f}{m [\ln(r_D)]}$$

Step 7. Calculate  $\frac{-J p_i t}{(W_e)_{\max}} \left[ 1 - \exp\left(-\frac{J p_i \Delta t_n}{(W_e)_{\max}}\right) \right]$

Step 8. Calculate the incremental water influx

$$(\Delta W_e)_n = \frac{(W_e)_{\max}}{p_i} [(\bar{p}_a)_{n-1} - (\bar{p}_r)_n] \left[ 1 - \exp\left(-\frac{J p_i \Delta t_n}{(W_e)_{\max}}\right) \right]$$

$$(\bar{p}_r)_n = \frac{(P_r)_n + (P_r)_{n-1}}{2} \quad (\bar{p}_a)_{n-1} = p_i \left( 1 - \frac{(W_e)_{n-1}}{(W_e)_{\max}} \right)$$

n	t days	P <sub>r</sub>	( $\bar{p}_r$ ) <sub>n</sub>	( $\bar{p}_a$ ) <sub>n-1</sub>	( $\bar{p}_a$ ) <sub>n-1</sub> - ( $\bar{p}_r$ ) <sub>n</sub>	( $\Delta W_e$ ) <sub>n</sub> MM bbl	(W <sub>e</sub> ) MM bbl
0	0	2740					
1	365	2500					
2	730	2290					
3	1095	2109					
4	1460	1949					

**Solution:**

Step 1. Calculate the reservoir radius  $r_e$

Area = 1 Acre = 43560 Square foot

multiply the area value by 43560

$$A = \pi \cdot r_e^2$$

$$r_e = \sqrt{A/\pi}$$

$$r_e = 9200 \text{ ft}$$

Step 2. Calculate the equivalent aquifer radius  $r_a$

$$r_a = 45793 \text{ ft}$$

Step 3. Calculate the dimensionless radius  $r_D$ .

$$r_D = r_a / r_e$$

$$r_D = 5$$

Step 4. Calculate initial water in place  $W_i$ .

$$W_i = \pi(r_a^2 - r_e^2) h \phi / 5.615$$

$$W_i = 28.15 \text{ MMMbbl}$$

Step 5. Calculate  $(W_e)_{\max}$

$$(W_e)_{\max} = c_t W_i p_i f$$

$$W_{e,\max} = 209,984,705 \text{ bbl}$$

Step 6. Calculate the productivity index  $J$  of the radial aquifer

$$J = \frac{0.00708 kh f}{m [\ln(r_D)]}$$

$$J = 62.4 \text{ bbl/day/psi}$$

Step 7. Calculate  $\frac{-J p_i t}{(W_e)_{\max}}$

$$\frac{-J p_i t}{(W_e)_{\max}} = 0.30$$

Step 7. Calculate  $\frac{-J p_i t}{(W_e)_{\max}}$   $\frac{-J p_i t}{(W_e)_{\max}}$  (0.30)

$$\left[ 1 - \exp\left(-\frac{J p_j \Delta t_n}{(W_e)_{\max}}\right) \right] \quad 0.26$$

Step 8. Calculate the incremental water influx

$$(\Delta W_e)_n = \frac{(W_e)_{\max}}{p_i} [(\bar{p}_a)_{n-1} - (\bar{p}_r)_n] \left[ 1 - \exp\left(-\frac{J p_j \Delta t_n}{(W_e)_{\max}}\right) \right]$$

$$(\Delta W_e)_1 = \frac{(W_e)_{\max}}{p_i} [p_i - (\bar{p}_r)_1] \left[ 1 - \exp\left(-\frac{J p_i \Delta t_1}{(W_e)_{\max}}\right) \right] \quad (\bar{p}_r)_1 = \frac{p_i + (p_r)_1}{2}$$

$$(\Delta W_e)_2 = \frac{(W_e)_{\max}}{p_i} [(\bar{p}_a)_1 - (\bar{p}_r)_2] \left[ 1 - \exp\left(-\frac{J p_i \Delta t_2}{(W_e)_{\max}}\right) \right] \quad (\bar{p}_a)_1 = p_i \left( 1 - \frac{(\Delta W_e)_1}{(W_e)_{\max}} \right)$$

$$(\bar{p}_r)_n = \frac{(p_r)_n + (p_r)_{n-1}}{2}$$

$$(\bar{p}_a)_{n-1} = p_i \left( 1 - \frac{(W_e)_{n-1}}{(W_e)_{\max}} \right)$$

$$p_i = 2,740 \text{ psi}$$

$$W_{e,\max} = 209,984,705 \text{ bbl}$$

$$(\bar{p}_r)_n = \frac{(p_r)_n + (p_r)_{n-1}}{2}$$

$$(\bar{p}_a)_{n-1} = p_i \left( 1 - \frac{(W_e)_{n-1}}{(W_e)_{\max}} \right)$$

$$p_i = 2,740 \text{ psi}$$

$$W_{e,\max} = 209,984,705 \text{ bbl}$$

n	Time,	Pe	(Pe)avg,n	(Pa)avg,n-1	(Pa)avg,n-1 - (Pe)avg,n	(Δwe)n	We
	days	psi	psi	psi		MMbbl	MMbbl
0	0	2740	2740	2740	0	0	0
1	365	2500	2620	2,740	120	2.4	2.36
2	730	2290	2395	2,709	314	6.2	8.55
3	1095	2109	2199.5	2,628	429	8.4	17.00
4	1460	1949	2029	2,518	489	9.6	26.64



### Test example

1. The initial reserves (OOIP and OGIP) at the time of the exploration is determined by:
  - a. Wait and weight method
  - b. Volumetric method
  - c. Drillers method
  - d. Cannot be calculated
2. After a few years of production from the reservoir the initial reserves volumes can be clarified. For these clarification calculations, which equation is used?
  - a. Tracy's equation
  - b. Havlena-Odeh material balance equation
  - c. Fetkovich's equations
  - d. Special finite material balance equations
3. In case there is no water influx ( $W_e=0$ ) the cumulative OOIP will be equal at each time step ( $N_1=N_2=N_3=...=N_j=...=N_n=N$ ).
  - a. True
  - b. False
4. Pair the following finite material balance equations with the reservoir where it is used for initial reserve calculation.
  - a. \_\_\_ Undersaturated oil reservoir with water influx
  - b. \_\_\_ Saturated oil reservoir if  $m \neq 0$ ,  $E_g \neq 0$  and  $E_{f,w} \neq 0$
  - c. \_\_\_ Dry gas reservoir
  - d. \_\_\_ Gas condensate reservoir with

water influx  $Y = G + U \frac{\sum \Delta p W_e D}{B_g - B_{gi}}$

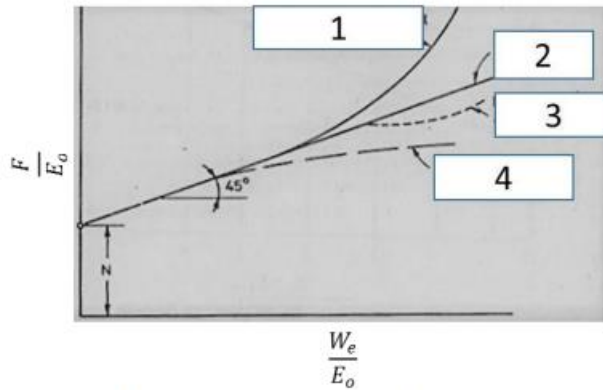
1.  $\frac{F}{E_o - E_g} = N + N(1 + m) \frac{E_g + E_{f,w}}{E_o - E_g}$
2.  $\frac{G_p B_g + W_p}{B_g - B_{gi}} = G + \frac{W_e}{B_g - B_{gi}}$
3.  $\frac{N_p B_o + W_p B_w}{B_{oi} c_{eff} \Delta p} = N + \frac{U \sum \Delta p W_D}{B_{wi} c_{eff} \Delta p}$
4.  $\frac{N_p B_o + W_p B_w}{B_{oi} c_{eff} \Delta p} = N + \frac{U \sum \Delta p W_D}{B_{wi} c_{eff} \Delta p}$

5. In case of an open undersaturated oil reservoir the correctly choose parameters

will result a linear if we graph  $\frac{N_p B_o + W_p B_w}{B_{oi} c_{eff} \Delta p}$  as a function of  $\frac{U \sum \Delta p W_D}{B_{wi} c_{eff} \Delta p}$ . What is the y intercept of this linear?

- a. U (B) aquifer constant
  - b.  $W_p$ , volume of produced water
  - c. N, the initial oil reserve
  - d.  $N_p$ , volume of produced oil
6. In case of a saturated closed reservoir we assume the there is an initial gas cap present and therefore, the expansion of gas needs to be accounted for. The expansion of formation and connate water is negligible. If we present the correct graph the slope will be N (the initial reserve nad the linear will go through the origin).
    - a. True
    - b. False

7. In case of a saturated open oil reservoir the graphical representation of the data will result the following diagram. Match the description with the numbers of the diagram.
- \_\_\_ High calculated water influx
  - \_\_\_ Incorrect aquifer geometry
  - \_\_\_ Low calculated water influx
  - \_\_\_ Correctly calculated water influx



8. If  $\frac{F}{E_o + mE_g}$  as the function of  $\frac{W_e}{E_o + mE_g}$  representing in case of an open saturated oil reservoir the desired graph is a linear with N (OOIP) as the y intercept. What is the slope of this correct linear?
- 35°
  - 40°
  - 45°
  - $\text{tg}\alpha = U$

9. In case of a dry gas reservoir the compressibility of the formation and the connate water is also very important.
- True
  - False
10. The correct calculation for gas condensate reservoir without water influx contains the moles volume, the molar weight and the density of the gas condensate in the system.
- True
  - False

## Sample of Midterm Assignments

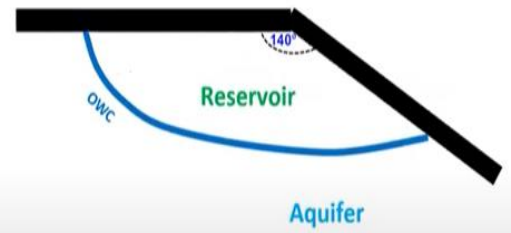
### Assignment 1

Using Fetkovich's method, calculate the water influx as a function of time for the following reservoir-aquifer and boundary pressure data:

$p_i = 2740$  psi       $h = 100'$        $c_t = 7 \times 10^{-6}$  psi  
 $\mu_w = 0.55$  cp       $k = 200$  md       $\theta = 140^\circ$   
 reservoir area = 40,363 acres      aquifer area = 1,000,000 acres.

Time, days	$p_r$ , psi
0	2740
365	2500
730	2290
1095	2109
1460	1949

$p_r$     reservoir pressure  
 $p_e$     external pressure



## Assignment 2

### Field Case of an Abnormally Pressured Gas Reservoir

This assignment addresses the analysis of an "abnormally pressured" gas reservoir (Reservoir "A"). You are to review these data and make predictions of total gas reserves (*i.e.*, gas-in-place) based on the "dry gas" and "high pressure gas" material balance equations.

#### Given:

*Production/Pressure History: Abnormally Pressured Gas Reservoir "A"*

Point	$p$ (psia)	$z$ -factor (dim-less)	$G_p$ (BSCF)	$p/z$ (psia)	$\frac{\bar{p}}{\bar{z}} [1 - \bar{c}_e(p_i - \bar{p})]$ (psia)
1	7200	1.184	0.00	6079	6079
2	6975	1.167	0.70	5976	5909
3	6721	1.148	1.53	5854	5714
4	6430	1.126	2.47	5708	5488
5	6233	1.112	3.33	5605	5334
6	6244	1.113	3.66	5611	5343
7	5937	1.091	4.27	5443	5099
8	5719	1.076	5.21	5317	4924
9	5347	1.050	6.07	5091	4619
10	4522	0.999	8.30	4527	3920
11	3671	0.956	10.62	3841	3163
12	3116	0.935	12.24	3331	2651
13	2835	0.928	13.17	3054	2388
14	2827	0.928	13.53	3046	2380
15	2773	0.927	13.76	2992	2330
16	2763	0.927	14.02	2982	2320

#### Required:

Analysis of  $p/z$  versus  $G_p$  Performance

*Theory: (Gas Material Balance Equations)*

• Dry Gas Case: (No Influx)

• High Pressure" Gas Material Balance Equation:

$$\frac{\bar{p}}{\bar{z}} = \frac{p_i}{z_i} \left[ 1 - \frac{G_p}{G} \right]$$

$$\frac{\bar{p}}{\bar{z}} = \frac{p_i}{z_i} \left[ \frac{1}{1 - \bar{c}_e(p_i - \bar{p})} \right] \left[ 1 - \frac{G_p}{G} \right]$$

*Tasks:*

1. Estimate the original gas-in-place ( $G$ ) and determine the "apparent" gas-in-place ( $G_{app}$ ) using the dry gas material balance equation.

Ans. 1.a  $G = \underline{\hspace{2cm} 24 \hspace{2cm}} \text{ BSCF}$

Ans. 1.b  $G_{app} = \underline{\hspace{2cm} 45 \hspace{2cm}} \text{ BSCF}$

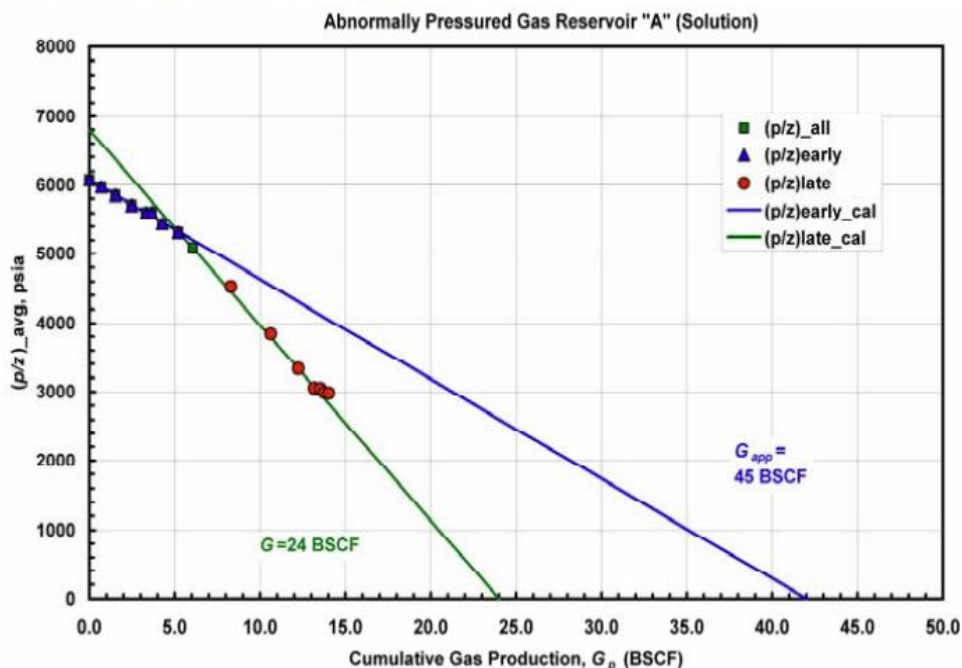
- 2 Assume that  $c_e(p) = 50 \times 10^{-6} \text{ psia}^{-1}$  — use the following relation to estimate the gas-in-place:

$$\frac{\bar{p}}{\bar{z}} [1 - \bar{c}_e(p_i - \bar{p})] = \frac{p_i}{z_i} \left[ 1 - \frac{G_p}{G} \right]$$

Ans. 2  $G = \underline{\hspace{2cm} 23 \hspace{2cm}} \text{ BSCF}$

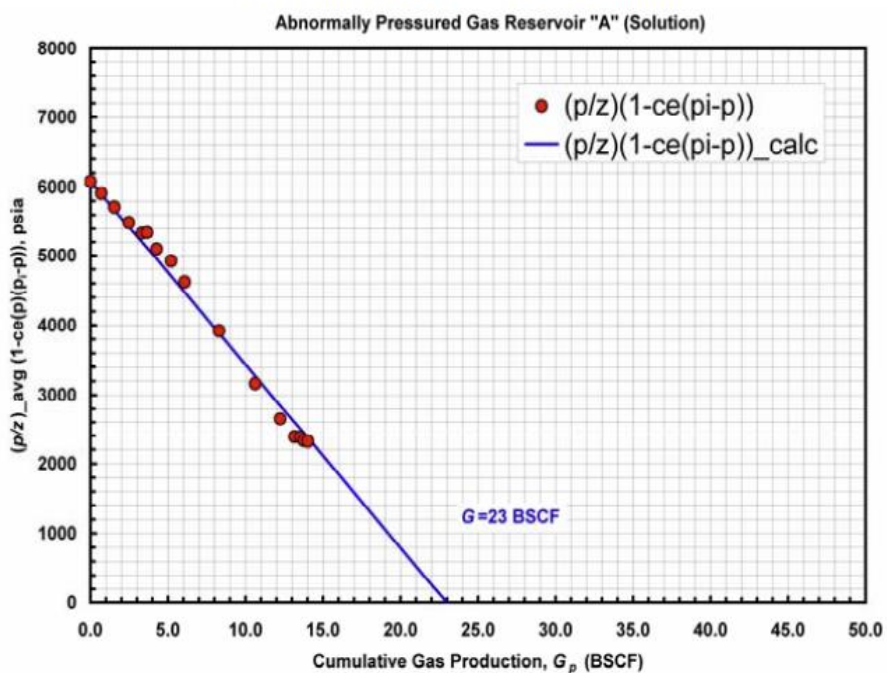
Field Case of an Abnormally Pressured Gas Reservoir (Continued)

*p/z* Versus Cumulative Gas Production ( $G_p$ ): Reservoir "A"



"Corrected" *p/z* Versus Cumulative Gas Production ( $G_p$ ): Reservoir "A"

(You must plot your results by hand)



### Assignment 3

A gas reservoir covers an area of  $5 \cdot 10^7 \text{ m}^2$ , with an average net thickness of 12 m and an average porosity of 26%. The irreducible water saturation in the reservoir is 21%. The initial pressure ( $p_i$ ) and temperature ( $T_i$ ) were 35100 kPa and 70 deg C, respectively. "Standard" conditions at the gas separator are  $p_{sc} = 200 \text{ kPa}$  and  $T_{sc} = 15.5 \text{ deg C}$ , for which the gas Z-factor ( $z_{sc}$ ) is 0.997. The table to the right reports gas Z-factors that have been measured in lab tests conducted over a broad range of pressures, and produced gas volumes measured early in the producing life of this reservoir.

Pressure (kPa)	Z-Factor	Gp (MMscm)
35100	0.914	0.0
34100	0.899	337.4
33100	0.886	714.4
32100	0.873	1091.5
31100	0.861	1528.1
30100	0.851	1984.6
29100	0.841	2480.7
28100	0.833	2976.8
27100	0.825	3532.5
26100	0.819	4108.0
25100	0.813	4723.3
24100	0.808	5318.6
23100	0.805	5953.7
22100	0.802	
21100	0.801	
20100	0.8	
19100	0.8	
18100	0.802	
17100	0.804	
16100	0.808	
15100	0.812	
14100	0.817	
13100	0.824	
12100	0.831	
11100	0.84	
10100	0.849	
9100	0.859	
8100	0.871	
7100	0.883	
6100	0.897	
5100	0.911	

- Using the volumetric method, estimate the gas initially in place (GIIP) in MMrcm (i.e., millions of reservoir cubic metres).
- Calculate the initial value of the gas expansion factor ( $E_i$ ).  
[Hint:  $E = (z_{sc}/z) \cdot (p/p_{sc}) \cdot (T_{sc}/T)$ ]
- Estimate gas initially in place ( $G$ ) at "standard" conditions (i.e., in MMscm).
- Assuming volumetric depletion, generate a plot of  $p/z$  against the fraction of gas initially in place that is produced (i.e.,  $G_{pd}$ ). I.E., calculate  $p/z$  using  $p/z = (p_i/z_i) \cdot (1 - G_{pd})$ , varying  $G_{pd}$  from 0 to 1. Using this line, determine  $G_{pd}$  and  $G_p$  at an abandonment pressure of 5100 kPa.
- The produced gas volumes reported in column 3 of the table are higher than those calculated assuming depletion (you can confirm this if you like by converting the produced volumes,  $G_p$ , to fraction of gas in place,  $G_{pd} = G_p/G$ , since you know  $G$  from question 3). This indicates that there is water influx. Assuming an aquifer model of the form  $W_e = c(p_i - p)$ , use the apparent gas-in-place method\*\* to estimate the value of the constant  $c$ , as well as the gas in place ( $G$ , in MMscm).
- Use your function for  $W_e$  to estimate (a) the fraction of gas initially in place and (b) the absolute volume that will be recovered at an abandonment pressure of 5100 kPa. Comment on how these compare to the results you obtained in question 4.

\*\* Procedure

For each pressure at which  $G_p$  is given:

- Calculate apparent gas in place,  $G_a$ . Recall that  $G_p = G_a \left(1 - \frac{E}{E_i}\right)$ ,  $\therefore G_a = G_p / \left(1 - \frac{E}{E_i}\right)$ .  $E$  denotes the gas expansion factor at the reservoir pressure of interest.
- Guess a value for the constant  $c$ , then calculate  $\frac{W_e E}{1 - E/E_i}$ .
- Graph  $G_a$  on the y-axis, against  $\frac{W_e E}{1 - E/E_i}$  on the x-axis. Recall that, when you have guessed the correct form of the aquifer function, the following relationship is supposed to hold true:  $G_a = G + \frac{W_e E}{1 - E/E_i}$ .
- Fit a linear trendline through your graph. Keep adjusting your guess for  $c$  until your data points fall on a line with slope  $\sim 1$ . When you have achieved this, you have found the appropriate value for  $c$ , and the y-intercept of your trendline represents an estimate of the gas in place ( $G$ ) in MMscm.

